EXERCICES DU CHAPITRE III

1 Exercice III-1

Etude de la fonction de Production Q en fonction du Capital K et du travail L
Q = f(K,L) = aK$^b L^c e^\epsilon \Rightarrow Log(Q) = bLog(K) + cLog(L) + Log(a) + \epsilon$
On dispose d'un échantillon de 10 entreprises de la même branche.
Modèle 1 MCO

$$\begin{aligned} \widehat{Log(Q)} &= 0,272Log(K) + 0,632Log(L) + 3,61 \\ &\quad (0,079) \qquad (0,076) \qquad (1,962) \end{aligned}$$

n=10 $R^2 = 0,94$ somme des carrés de résidus SCR_1=1,8569 DW=2,993

Les chiffres entre parenthèses sont les écart-type estimés des coefficients
Modèle 2 MCO

$$\begin{aligned} \widehat{Log(\frac{Q}{L})} &= 0,272Log(\frac{K}{L}) - 0,0953Log(L) + 3,61 \\ &\quad (0,079) \qquad (0,1424) \qquad (1,962) \end{aligned}$$

n=10 $R^2 = 0,94$ $SCR_2 = 1,8569$ DW=2,993

Modèle 3 MCO

$$\begin{aligned} \widehat{Log(\frac{Q}{L})} &= 0,321Log(\frac{K}{L}) + 2,303 \\ &\quad (0,029) \qquad (0,173) \end{aligned}$$

n=10 $R^2 = 0,938$ $SCR_3 = 1,9757$ DW=2,84

1) Dans le modèle 1, $Log(Q) = bLog(K) + cLog(L) + Log(a) + \epsilon$
On veut tester l'hypothèse $H_0 : b + c = 1$ contre $H_1 : b + c \neq 1$
a) montrer que la contrainte H_0 conduit au modèle 3
b) construire le test
2) Montrer que le test de nullité de l'un des coefficients du modèle 2 permet de tester l'hypothèse de la question 1); Faire ce test.
3) Comparer ces trois modèles.

2 Exercice III-2

Equation d'Investissement : données trimestrielles sur la période 1966-1981
Les données Investissement I, Capital K et PIB Y sont en volume au prix 1970.

1) Période 1966-1 1981-4 n=64 $R^2 = 0{,}991$ DW+1.77 s=416,21 les chiffres entre parenthèses sont les "t de Student" dans toute la suite

$$\begin{array}{rcl} I_t & = & 0,913 I_{t-1} + 0,178 Y_t - 0,170 Y_{t-1} + 750 \\ & & (21,11) \quad (10{,}85) \quad (\text{-}9{,}90) \quad (1{,}1) \end{array}$$

Etudier l'autocorrélation de ce modèle.
2) On note e_t les résidus et on construit les trois équations suivantes

$$\begin{array}{rcl} e_t & = & 0,0027 Y_t - 249 \\ & & (5,51) \quad (\text{-}0.5) \end{array}$$

$R^2 = 0,51$ s=137

$$\begin{array}{rcl} e_t & = & 2,43\sqrt{Y_t} - 779 \\ & & (3,51) \quad (\text{-}1.1) \end{array}$$

$R^2 = 0,17$ s=237

$$\begin{array}{rcl} e_t & = & 0,00034 K_t - 181 \\ & & (3,57) \quad (\text{-}0.8) \end{array}$$

$R^2 = 0,51$ s=136
Montrer qu'il y a hétéroscédasticité du modèle 1. Comment peut-on traiter cette hétéroscédasticité? (plusieurs possibilités).

3) Tester l'hétéroscédasticité à l'aide du test de Goldfeld et Quandt.
Période 1966-1 1972-4 n=28 $R^2 = 0{,}995$ s=306,76

$$\begin{array}{rcl} I_t & = & 1,04 I_{t-1} + 0,169 Y_t - 0,181 Y_{t-1} + 1007 \\ & & (10,44) \quad (12{,}77) \quad (\text{-}9{,}82) \quad (2{,}61) \end{array}$$

Période 1975-1 1981-4 n=28 $R^2 = 0{,}932$ s=521,32

$$\begin{array}{rcl} I_t & = & 0,784 I_{t-1} + 0,195 Y_t - 0,164 Y_{t-1} - 747 \\ & & (4,53) \quad (2{,}84) \quad (\text{-}2{,}04) \quad (\text{-}0{,}4) \end{array}$$

3 Exercice III-3

On étudie l'évolution du salaire horaire trimestriel dans le secteur marchand.
TW Le taux de croissance du salaire horaire
TP le taux de croissance de l'indice des prix à la consommation
TCHO le taux de chômage
TSMIC le taux de croissance du SMIC
DU821 une variable muette au premier trimestre de 1982

1) Etudier la normalité des erreurs du modèle 1

```
Linear Regression - Estimation by Least Squares
Dependent Variable TW
Quarterly Data From 71:01 To 90:02
Usable Observations     78      Degrees of Freedom    73
Centered R**2     0.898940      R Bar **2   0.893402
Uncentered R**2   0.983631      T x R**2      76.723
Mean of Dependent Variable      2.7127431308
Std Error of Dependent Variable 1.2003302451
Standard Error of Estimate      0.3918991649
Sum of Squared Residuals        11.211701746
Regression F(4,73)                  162.3358
Significance Level of F           0.00000000
Log Likelihood                     -35.02692
Durbin-Watson Statistic              1.531488

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
1.  Constant                 2.028374490  0.202618992     10.01078  0.00000000
2.  TCHO                    -0.142141667  0.017619065     -8.06749  0.00000000
3.  TP                       0.585284028  0.068695111      8.52002  0.00000000
4.  TSMIC                    0.154295514  0.044394795      3.47553  0.00086199
5.  DU821                    1.970459744  0.401775623      4.90438  0.00000551
```

Skewness $= \widehat{\alpha_3} = 0,092$
Kurtosis $= \widehat{\alpha_4} = 3,045$

2) Etudier l'autocorrélation dans ce modèle 1
TEST DE DURBIN ET WATSON
DW= 1.53149
TEST DE LJUNG-BOX
statistique Q(17)= 17.29204 niveau de significativité 0.4348
3) Faire le test de White. Conclusion.

```
Linear Regression - Estimation by Least Squares
Dependent Variable RES^2
Quarterly Data From 71:01 To 90:02
Usable Observations     78      Degrees of Freedom    68
Centered R**2     0.055268      R Bar **2  -0.069770
Uncentered R**2   0.373766      T x R**2      29.154
Mean of Dependent Variable      0.1437397660
Std Error of Dependent Variable 0.2028588738
Standard Error of Estimate      0.2098162859
Sum of Squared Residuals        2.9935554206
Regression F(9,68)                    0.4420
Significance Level of F           0.90731317
Log Likelihood                     16.47243
Durbin-Watson Statistic            2.186611

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
1.  Constant                 0.060087860  0.306120669      0.19629  0.84497007
2.  TCHO                    -0.007931965  0.067466006     -0.11757  0.90675512
3.  TP                      -0.052724850  0.211261043     -0.24957  0.80367069
4.  TSMIC                    0.070911167  0.110965428      0.63904  0.52494477
5.  TCHO^2                   0.000735336  0.005152779      0.14271  0.88694389
6.  TP^2                     0.034126473  0.049355060      0.69145  0.49163785
7.  TSMIC^2                  0.003665925  0.015376137      0.23842  0.81227506
8.  TCHO*TP                  0.001248912  0.017263177      0.07235  0.94253937
9.  TCHO*TSMIC              -0.000069305  0.010828840     -0.00640  0.99491230
10. TP*TSMIC                -0.032101729  0.041897634     -0.76619  0.44621235
```

4) Etudier la stabilité du modèle

```
**************************************************************
sous-periodes utilisees 71:01 81:02 81:03 90:02
F(4,70)=      0.96636 with Significance Level 0.43153506
**************************************************************
sous-periodes utilisees 71:01 81:03 81:04 90:02
F(4,70)=      1.10844 with Significance Level 0.35951177
**************************************************************
sous-periodes utilisees 71:01 81:04 82:01 90:02
F(4,70)=      0.88558 with Significance Level 0.47720928
**************************************************************
sous-periodes utilisees 71:01 82:01 82:02 90:02
F(4,70)=      3.04239 with Significance Level 0.02263533
**************************************************************
sous-periodes utilisees 71:01 82:02 82:03 90:02
F(4,70)=      4.12216 with Significance Level 0.00467526
**************************************************************
sous-periodes utilisees 71:01 82:03 82:04 90:02
F(4,70)=      3.20333 with Significance Level 0.01787107
**************************************************************
sous-periodes utilisees 71:01 82:04 83:01 90:02
F(4,70)=      2.92609 with Significance Level 0.02685351
**************************************************************
sous-periodes utilisees 71:01 83:01 83:02 90:02
F(4,70)=      3.39429 with Significance Level 0.01350569
**************************************************************
sous-periodes utilisees 71:01 83:02 83:03 90:02
F(4,70)=      2.43673 with Significance Level 0.05506805
**************************************************************
sous-periodes utilisees 71:01 83:03 83:04 90:02
F(4,70)=      2.21984 with Significance Level 0.07558402
**************************************************************
sous-periodes utilisees 71:01 83:04 84:01 90:02
F(4,70)=      2.06189 with Significance Level 0.09506955
**************************************************************
```

4 Exercice III-4

On explique Y en fonction de x_1 et x_2
MCO sur une période de n=50 trimmestres

$$\begin{array}{rcl} y & = & 1,83x_1 + 1,51x_2 - 391,10 \\ & & (2,45) \quad (3,97) \quad (\text{-}4,42) \end{array}$$

n=50 $R^2 = 0,766$ DW = 0,328 s = 54,472

MCO sur les 25 premiers trimestres

$$\begin{array}{rcl} y & = & 0,77x_1 - 0,40x_2 + 30,93 \\ & & (6,09) \quad (\text{-}2{,}12) \quad (7{,}86) \end{array}$$

n=25 $R^2 = 0,979$ DW = 1,94 s = 1,22

MCO sur les 25 derniers trimestres

$$\begin{array}{rcl} y & = & 0,21x_1 + 0,41x_2 + 25,34 \\ & & (7,71) \quad (11{,}78) \quad (2{,}59) \end{array}$$

n=25 $R^2 = 0,987$ DW =2,04 s = 1,037

Résultats sur le modèle transformé de la méthode d'Hildreth-Lu

$\widehat{\rho}$	R^2	s	DW
0,5	0,7	34,7	0,81
0,6	0,67	32,1	1,02
0,7	0,61	30,2	1,25
0,8	0,55	29,3	1,3
0,9	0,50	29,6	1,32

1) Etudier les propriétés des erreurs du modèle 1

2) Exposer et effectuer la méthode d'Hildreth-Lu

3) Etudier la stabilité de ce modèle 1. Conclusion.

5 Exercice III-5

Etude du taux d'intérêt à 3 mois TBILL en fonction du taux d'intérêt à 3 ans R3 et du taux à 10ans R10
1) comparer ces trois modèles

```
Linear Regression - Estimation by Least Squares                MODELE 1
Quarterly Data From 60:01 To 91:04
Usable Observations    128      Degrees of Freedom   125
Centered R**2     0.919547      R Bar **2   0.918260
Uncentered R**2   0.987211      T x R**2     126.363
Mean of Dependent Variable      6.3959375000
Std Error of Dependent Variable 2.7915105942
Standard Error of Estimate      0.7980987958
Sum of Squared Residuals        79.620210986
Regression F(2,125)                 714.3532
Significance Level of F           0.00000000
Log Likelihood                    -151.23935
Durbin-Watson Statistic             0.324149
Q(32)                           269.5   Niveau de significativite 0.000

   Variable                     Coeff       Std Error      T-Stat     Signif
```

```
*****************************************************************************
1.  Constant                -0.668961589  0.200301063     -3.33978  0.00110555
2.  R3                       0.961161359  0.025584819     37.56764  0.00000000
3.  DU823                   -2.000796713  0.813750058     -2.45874  0.01531146


Linear Regression - Estimation by Least Squares                 MODELE 2
Dependent Variable TBILL
Quarterly Data From 60:01 To 91:04
Usable Observations    128      Degrees of Freedom   125
Centered R**2     0.829302      R Bar **2   0.826570
Uncentered R**2   0.972866      T x R**2     124.527
Mean of Dependent Variable      6.3959375000
Std Error of Dependent Variable 2.7915105942
Standard Error of Estimate      1.1625209673
Sum of Squared Residuals        168.93187493
Regression F(2,125)                 303.6430
Significance Level of F           0.00000000
Log Likelihood                    -199.38191
Durbin-Watson Statistic             0.280622
Q(32)                           94.64    Niveau de significativite 0.000

   Variable                     Coeff       Std Error      T-Stat     Signif
*****************************************************************************
1.  Constant                -0.675046907  0.305751137     -2.20783  0.02908040
2.  R10                      0.928585025  0.037937943     24.47642  0.00000000
3.  DU823                   -1.797988618  1.185817753     -1.51624  0.13198260


Linear Regression - Estimation by Least Squares                 MODELE 3
Quarterly Data From 60:01 To 91:04
Usable Observations    128      Degrees of Freedom   124
Centered R**2     0.965651      R Bar **2   0.964820
Uncentered R**2   0.994540      T x R**2     127.301
Mean of Dependent Variable      6.3959375000
Std Error of Dependent Variable 2.7915105942
Standard Error of Estimate      0.5235880740
Sum of Squared Residuals        33.993914434
Regression F(3,124)                1161.9878
Significance Level of F           0.00000000
Log Likelihood                     -96.76981
Durbin-Watson Statistic             0.688355
Q(32)                           185.81   Niveau de significativite 0.000

   Variable                     Coeff       Std Error      T-Stat     Signif
*****************************************************************************
1.  Constant                -0.012846885  0.140904917     -0.09117  0.92750134
2.  R3                       2.253570453  0.101576633     22.18591  0.00000000
3.  R10                     -1.334009658  0.103404898    -12.90084  0.00000000
4.  DU823                   -1.800933839  0.534080735     -3.37203  0.00099567
```

2) Avec les résultats suivants et les résultats ci-dessus étudier en détail les propriétés des erreurs et des coefficients du modèle 3.
Coefficient de symétrie Skewness 0,35
Cofficient d'applatissement Kurtosis 3,086

```
Linear Regression - Estimation by Least Squares
Dependent Variable RES^2
Quarterly Data From 60:01 To 91:04
Usable Observations    128      Degrees of Freedom   122
Centered R**2     0.214405      R Bar **2   0.182208
Uncentered R**2   0.473096      T x R**2      60.556
Mean of Dependent Variable      0.2655774565
Std Error of Dependent Variable 0.3805125169
Standard Error of Estimate      0.3441045243
Sum of Squared Residuals        14.445766683
```

```
Regression F(5,122)                  6.6592
Significance Level of F           0.00001620
Log Likelihood                     -41.99989
Durbin-Watson Statistic             1.533835

   Variable                    Coeff       Std Error      T-Stat     Signif
*******************************************************************************
1.  Constant                -0.118596433  0.262912055     -0.45109  0.65272700
2.  R3                      -0.304275812  0.278022764     -1.09443  0.27592349
3.  R10                      0.352908968  0.306998280      1.14955  0.25257903
4.  R3^2                     0.135510588  0.128405302      1.05533  0.29335764
5.  R10^2                    0.078171249  0.140345114      0.55699  0.57855259
6.  R3*R10                  -0.213542362  0.266784114     -0.80043  0.42501688
```

```
*************************************************************
sous-periodes utilisees 60:01 80:04 81:01 91:04
F(3,121)=      8.25404 with Significance Level 0.00004847
*************************************************************
sous-periodes utilisees 60:01 81:01 81:02 91:04
F(3,121)=     13.78107 with Significance Level 0.00000009
*************************************************************
sous-periodes utilisees 60:01 81:02 81:03 91:04
F(3,121)=     17.47421 with Significance Level 0.00000000
*************************************************************
sous-periodes utilisees 60:01 81:03 81:04 91:04
F(3,121)=     13.64791 with Significance Level 0.00000010
*************************************************************
sous-periodes utilisees 60:01 81:04 82:01 91:04
F(3,121)=      9.87359 with Significance Level 0.00000713
*************************************************************
sous-periodes utilisees 60:01 82:01 82:02 91:04
F(3,121)=      8.15856 with Significance Level 0.00005435
*************************************************************
```

3) Etudier les deux sous-modèles proposés.

```
Linear Regression - Estimation by Least Squares
Dependent Variable TBILL
Quarterly Data From 60:01 To 81:02
Usable Observations     86      Degrees of Freedom    83
Centered R**2     0.970032      R Bar **2   0.969309
Uncentered R**2   0.994301      T x R**2      85.510
Mean of Dependent Variable      5.6745348837
Std Error of Dependent Variable 2.7658338417
Standard Error of Estimate      0.4845392469
Sum of Squared Residuals        19.486597386
Regression F(2,83)                 1343.2877
Significance Level of F           0.00000000
Log Likelihood                     -58.19004
Durbin-Watson Statistic             0.789023
Q(21)                           77.34     niveau de significativite 0.000
   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
1.  Constant                -0.564489899  0.171702707     -3.28760  0.00148279
2.  R3                       2.325576878  0.134677599     17.26773  0.00000000
```

```
3.  R10                     -1.315177624  0.143473755     -9.16668  0.00000000


Linear Regression - Estimation by Least Squares
Dependent Variable RES^2
Quarterly Data From 60:01 To 81:02
Usable Observations     86      Degrees of Freedom    80
Centered R**2     0.095525      R Bar **2   0.038995
Uncentered R**2   0.506755      T x R**2      43.581
Mean of Dependent Variable      0.2265883417
Std Error of Dependent Variable 0.2496126459
Standard Error of Estimate      0.2446974063
Sum of Squared Residuals        4.7901456529
Regression F(5,80)                   1.6898
Significance Level of F           0.14657482
Log Likelihood                       2.14610
Durbin-Watson Statistic            1.452803

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
1.  Constant                -0.280445328  0.311355612     -0.90072  0.37043890
2.  R3                      -0.153518412  0.350044411     -0.43857  0.66215641
3.  R10                      0.272990388  0.417256819      0.65425  0.51482703
4.  R3^2                    -0.086913030  0.145921373     -0.59562  0.55311346
5.  R10^2                   -0.104727696  0.166100973     -0.63051  0.53016046
6.  R3*R10                   0.185990993  0.307060726      0.60571  0.54641933

Regression with AR1 - Estimation by Hildreth-Lu Search
Dependent Variable TBILL
Quarterly Data From 60:02 To 81:02
Usable Observations     85      Degrees of Freedom    81
Centered R**2     0.983000      R Bar **2   0.982370
Uncentered R**2   0.996768      T x R**2      84.725
Mean of Dependent Variable      5.6949411765
Std Error of Dependent Variable 2.7757284001
Standard Error of Estimate      0.3685529279
Sum of Squared Residuals        11.002332114
Regression F(3,81)                 1561.2258
Significance Level of F           0.00000000
Log Likelihood                     -33.71666
Durbin-Watson Statistic             1.759562
Q(21-1)                            31.851034
Significance Level of Q           0.04491172

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
1.  Constant                -0.537899854  0.426150190     -1.26223  0.21048800
2.  R3                       1.821376773  0.172927657     10.53259  0.00000000
3.  R10                     -0.822621212  0.205841144     -3.99639  0.00014100
*******************************************************************************
4.  RHO                      0.726467418  0.078598026      9.24282  0.00000000
*************************

Linear Regression - Estimation by Least Squares
Dependent Variable TBILL
Quarterly Data From 81:03 To 91:04
Usable Observations     42      Degrees of Freedom    38
Centered R**2     0.979156      R Bar **2   0.977511
Uncentered R**2   0.998492      T x R**2      41.937
Mean of Dependent Variable      7.8730952381
Std Error of Dependent Variable 2.2252174332
Standard Error of Estimate      0.3337014420
Sum of Squared Residuals        4.2315527919
Regression F(3,38)                 595.0363
Significance Level of F           0.00000000
Log Likelihood                     -11.39831
Durbin-Watson Statistic             1.223160
Q(10)                              12.27     niveau de significativite 0.267
   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
```

```
1.  Constant                 1.448502605  0.296684094      4.88231  0.00001916
2.  R3                       2.090235794  0.123547328     16.91850  0.00000000
3.  R10                     -1.333631375  0.136164471     -9.79427  0.00000000
4.  DU823                   -1.163495994  0.346969123     -3.35331  0.00181853


Linear Regression - Estimation by Least Squares
Dependent Variable RES^2
Quarterly Data From 81:03 To 91:04
Usable Observations     42      Degrees of Freedom    36
Centered R**2     0.169814      R Bar **2   0.054511
Uncentered R**2   0.456666      T x R**2      19.180
Mean of Dependent Variable      0.1007512569
Std Error of Dependent Variable 0.1403420640
Standard Error of Estimate      0.1364633966
Sum of Squared Residuals        0.6704013097
Regression F(5,36)                    1.4728
Significance Level of F           0.22285875
Log Likelihood                      27.29310
Durbin-Watson Statistic             1.353283

   Variable                     Coeff       Std Error      T-Stat     Signif
********************************************************************************
1.  Constant                -0.847010923  1.035057293     -0.81832  0.41855682
2.  R3                      -0.437501573  0.438871238     -0.99688  0.32547694
3.  R10                      0.608494603  0.586591670      1.03734  0.30649550
4.  R3^2                     0.056341004  0.106276354      0.53014  0.59927200
5.  R10^2                    0.006453267  0.138997182      0.04643  0.96322626
6.  R3*R10                  -0.070566375  0.240200551     -0.29378  0.77061234

Regression with AR1 - Estimation by Hildreth-Lu Search
Dependent Variable TBILL
Quarterly Data From 81:04 To 91:04
Usable Observations     41      Degrees of Freedom    36
Centered R**2     0.978772      R Bar **2   0.976414
Uncentered R**2   0.998768      T x R**2      40.949
Mean of Dependent Variable      7.6970731707
Std Error of Dependent Variable 1.9343038074
Standard Error of Estimate      0.2970681063
Sum of Squared Residuals        3.1769805526
Regression F(4,36)                  414.9721
Significance Level of F           0.00000000
Log Likelihood                      -5.74484
Durbin-Watson Statistic             1.824022
Q(10-1)                             8.341091
Significance Level of Q           0.50017124

   Variable                     Coeff       Std Error      T-Stat     Signif
********************************************************************************
1.  Constant                 1.651690805  0.471281930      3.50468  0.00124264
2.  R3                       1.776680404  0.178882038      9.93213  0.00000000
3.  R10                     -1.061930625  0.193014326     -5.50182  0.00000322
4.  DU823                   -0.867718188  0.274008176     -3.16676  0.00313513
********************************************************************************
5.  RHO                      0.493394423  0.135507088      3.64110  0.00084667
```